Physics 101 Chapter 2 & Lecture 2-3 1D kinematics

Equations

$$\frac{v - v_0}{\Delta t} = a$$

$$x = (\frac{v + v_0}{2})\Delta t$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0{}^2 + 2ax$$

Displacement and distance

A vector quantity means a quantity with both direction and magnitude. A scalar quantity means only magnitude. For example, 50 km/hr [W] is a vector whereas 50km /hr is a scalar.

Displacement is a vector quantity whereas distance is a scalar. In 1D kinematics, we will simply use positive and negative signs to denote direction.

Displacement is the change in position, where is also a vector quantity. Position is denoted by $\vec{x}$ (used in the textbook and lecture). Thus, displacement is $\Delta\vec{x}$, and is defined as $\vec{x} - \vec{x}_0$, where $\vec{x}_0$ is the initial position. For the sake of simplicity, the arrow will be omitted and is understood to be there. Distance is represent by x (without a direction arrow), and the change in distance (which is also distance), is denoted by Δx.

Due the vector quantity of displacement, distance is always equal or larger than the displacement in terms of magnitude.

We will now define average velocity and average speed

Average velocity and average speed

There always have been a misconception about the term average. We always think of it as the average of two values. However, this is not the case here!

Average speed is strictly defined as $\frac{\text{distance traveled}}{\text{elapsed time}}$.

Average velocity is strictly defined as $\frac{\textit{displacement}}{\text{elapsed time}}$, which becomes $\frac{\Delta x}{\Delta t}$. Note that Δ means change and it is always final quantity – initial quantity.

When and only when acceleration is constant, we can use the familiar method, which is $\frac{1}{2}(v+v_0)$ to find the average velocity. Using this method with a non constant acceleration will give you the wrong answer! The average velocity has the same direction as displacement since those two are the only ones with a direction.

Acceleration:
Acceleration, $\vec{a}$, is always a vector quantity. Which means that it has a direction associated with it! Unless they strictly ask you for the magnitude of the acceleration, it is always safer to include a sign.

Acceleration is defined as the $\frac{\text{change in velocity}}{\text{elapsed time}}$, which is $\frac{\Delta v}{\Delta t}$. When dealing with only speeds sometimes, we can define acceleration as $\frac{\text{change in speed}}{\text{elapsed time}}$.

Sometimes, we like to confuse people by talking about deacceleration. Deacceleration simply means slowing down! It does not mean that acceleration is negative.

In general, if velocity and acceleration have opposite signs (or velocity x acceleration < 0) then the object is slowing down. If velocity and acceleration have the same signs (velocity x acceleration > 0), the object is said to be speeding up.

Instantaneous velocity, speed, acceleration
Instantaneous simply means the quantity at a particular moment in time and not over a time interval.

Instantaneous velocity/speed is defined as
$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$, which simply means as the change in position/distance as time interval becomes infinitesimally small, which becomes an instant in time.

Instantaneous acceleration is defined as
$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$, which simply means as the change in velocity/speed as time interval becomes infinitesimally small, which becomes an instant in time.

Most of the time, when we talk about speed/velocity or acceleration, we are talking about the instantaneous speed/velocity and acceleration.

Kinematics Equations

In a special case where the **acceleration is constant**, we can derive 4 very useful kinematics formulas. Allow me to emphasize the key term "acceleration is constant". These formulas will ONLY APPLY WHEN THE ACCELERATION IS CONSTANT. Do not use them when acceleration is not constant or you will get the wrong answer. Constant acceleration means the acceleration is either a fixed value or 0.

The first equation is pretty straightforward. We know $\frac{v - v_0}{\Delta t} = a$, we can write this in another form, v = $v_0 + a\Delta t$

(Equation 1)

$$\boxed{\frac{v - v_0}{\Delta t} = a \text{ or } v = v_0 + a\Delta t}$$

Since we are dealing with constant acceleration here, we can write average velocity/speed = $\frac{1}{2}(v + v_0)$. We know that average velocity/speed x time give you displacement/distance.

Thus,

v Δt = x

sub in v = $\frac{1}{2}(v + v_0)$, we have

x = $\frac{1}{2}(v + v_0)\ \Delta t$

(Equation 2)

$$\boxed{x = \frac{1}{2}(v + v_0)\ \Delta t}$$

In equation 1, we said v = $v_0 + a\Delta t$. This, we can sub this into equation 2

v = $v_0 + a\Delta t$

x = $\frac{1}{2}(v + v_0)\ \Delta t$

becomes,

x= $\frac{1}{2}(v_0 + a\Delta t + v_0)\ \Delta t$

add the two v_0 together,

x= $\frac{1}{2}(2v_0 + a\Delta t)\ \Delta t$

Multiply them in, and get

x= $v_0 \Delta t + \frac{1}{2} a \Delta t^2$

(Equation 3)

$$x = v_0 \Delta t + \frac{1}{2} a \Delta t^2$$

Equation 4, which we will omit the derivation since it is pretty easy to remember, is simply

(Equation 4)

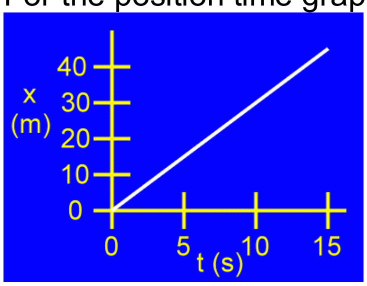
$$v^2 = v_0{}^2 + 2ax$$

In each equation, you have 4 variables. All the kinematics question will give you the values for 3 out of the 4 variables. A good strategy is to list all the variables you are given, and pick the appropriate equation that will give you the value for the 4th variable.

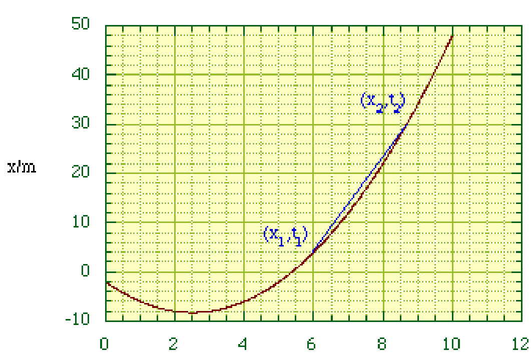
Graphical Analysis

There are three types of graphs, positive vs time, velocity vs time, and acceleration vs time.

For the position time graph, it could be either a straight line or a curved line.



(straight line)



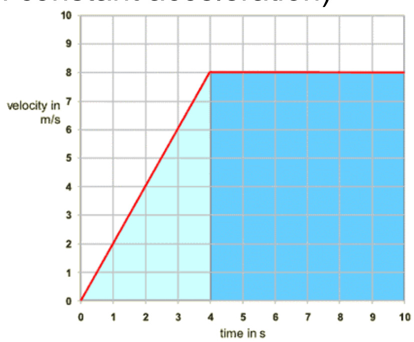
(Curved line)

The slope of a position –time graph is velocity since slope is measured as $\frac{x_2 - x_1}{t_2 - t_1}$, which is simply $\frac{\Delta x}{\Delta t}$.

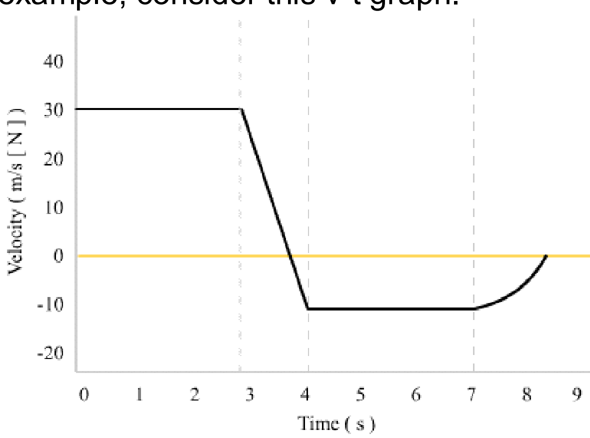
A position-time graph with a straight line has a constant velocity (slope remains the same). This means that the acceleration is zero.

A position-time graph with a curved line has a changing velocity. It means the acceleration is constant (non zero) (We are not considering non constant acceleration in phys101) To find the slope at point, we simply draw a tangent to that point and find the slope of that tangent. This will give us the velocity.

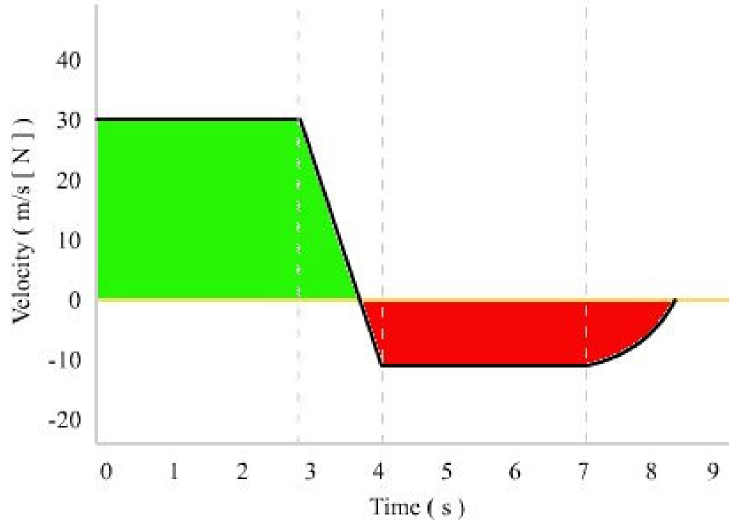
Velocity –time graph for this course is a straight line. (Non straight line velocity time graph has a non constant acceleration)



The slope give you acceleration, while the area under the velocity-time graph give you displacement. (from the graph to the x-axis) When they ask you the displacement form a velocity-time graph, they can play tricks on you! For example, consider this v-t graph.

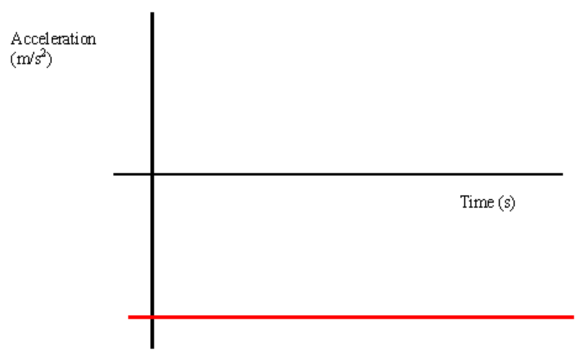


They ask you for the displacement. So seems simple enough, we just find the area under the graph. However, many students get wrong here!
Lets take a better look:

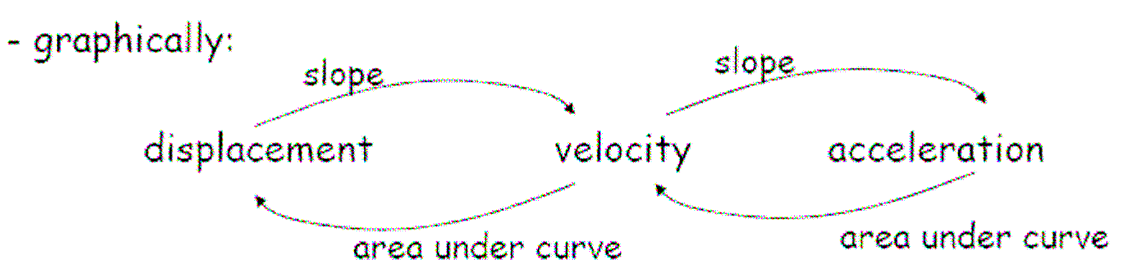


Here is the same graph but I have colored the area under the v-t graph. You must remember to use the green area – the red area because we are talking about displacement here, which has a direction! Green area + red area will give you the distance traveled but not the displacement.

Acceleration-time graph for this course is either a = 0 or a = some constant number. The area under the acceleration time graph give you velocity.



So to summarize the graphical analysis, here is a neat chart.



Free-falling bodies:
The key idea of free-falling bodies is that they all fall at the same rate (acceleration) and if they are near Earth's surface, they will fall at a rate of 9.8 m/s^2

Here is the reason why. We know that gravity acts stronger on a more massive object since $F_{gravity} = mg$, thus, higher mass = bigger force of gravity.

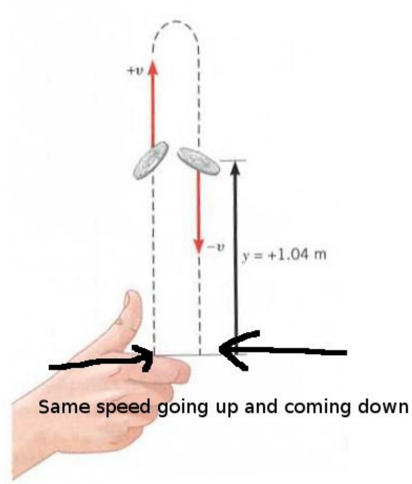
However, bigger mass also have bigger inertia (tendency to resist motion). If you think about it, it is harder to push a car than say a ball simply because the car is more massive and more able to resist motion better.

Well, gravity pulling on an object is like you pushing a car. Although gravity pulls harder on an object, the object also has a bigger inertia. Thus, these two balances out. As a result, all objects have a constant acceleration or rate of falling in a gravitational field (the place where free fall occurs) regardless of their mass or shape.

The assumption we make is that air resistance is negligible. This assumption is the foundation of free-fall. Without this assumption, air resistance will place a part and shapes will determine which objects fall faster.

Keep in mind that free falling does not necessarily mean that an object is falling down. Rather**, it is an object either moving upward or downward under the influence of gravity alone**.

Situations with involving tossing something up and let it undergo free fall have a time and speed symmetry. What I mean by speed symmetry this is that the speed coming down at the place where you toss the object is the same as the speed you tossed it. Allow me to demonstrate:



The reason for this that you lose g (9.8) as you go up and you regain g as you come down. Thus, speed will be the same. However, the velocity is not the same. The two speeds have different directions and thus different velocity. Thus, by knowing the speed of the object as it fall down at the position where you released it, you can figure out the release speed.

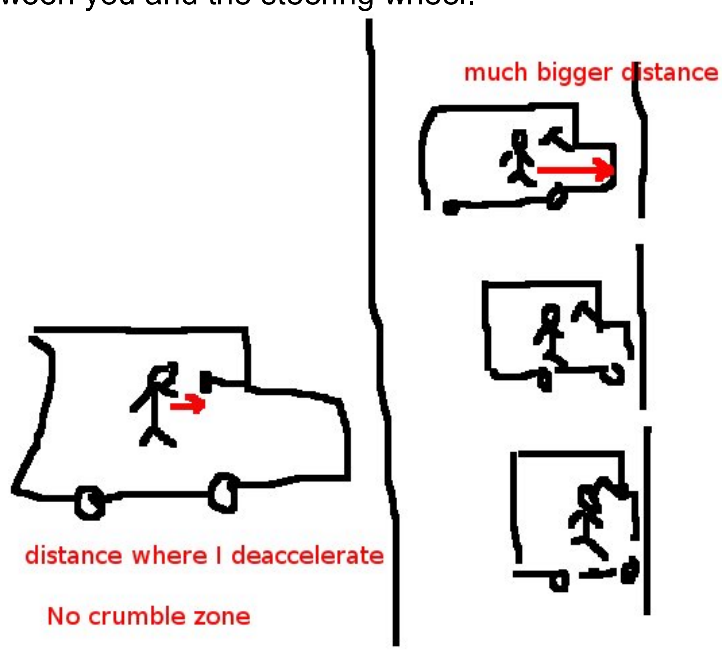
Time symmetry means that the time it takes to go up is the same as the time it takes to go down. This should be intuitive if you apply the same reason as in the speed symmetry.

For free-falling objects, we can apply the same 4 kinematics equations we derived earlier.

Real world physics (Airbag)
The physics of airbag was introduced in this lecture. Basically, it has three functions:

a) expands the crumble zone.
   Let's think about this. All the cars are designed to crumble on collision with say a wall. The reason for this is that the "deacceleration" would take place over a larger time interval.

   In a car with no crumble zone, it would hit the wall and stay rigid. As a result, you change your velocity from say 80 km/hr to 0 in the distance between you and the steering wheel.



However, in a car with crumble zone, your “deacceleration” distance is extended. This give will you a smaller acceleration. Consider the equation, $v^2 = {v_0}^2 + 2ax$. Solving for a, we have $a = \frac{v^2 - {v_0}^2}{2x}$, we know that v and v_0 stays constant. Thus, if we increase x, (divide by a bigger) number, we will have a smaller acceleration. Smaller acceleration means bigger force felt since F = ma, where m is the mass of your body and stays constant.

Now, we understand crumble zone, we can consider how airbag extends the crumble zone. Well, even with crumble zone, you will still hit the steering wheel hard. With airbag, your collision with the steering wheel is significantly reduced. Think of the airbag connecting you to the crumble zone. Without it, you will bang on the steering wheel hard after the crumble zone crumbles. With airbag, you are connected to the crumble of the car and thus will go with the “flow” rather than bang it.

b) Of course, the airbag also increases the distance for you to “deacceleration”, and there by reducing your acceleration and hence the force you feel significantly.
c) Airbags expands and fill a big area. By hitting the airbag rather than the smaller steering wheel, you are spreading the force you feel over a much larger area and thus you will get hurt less.